Ph.D. in Information Technology
Thesis Defense

October 24th, 2024
at 15:00
Aula Magna Rettorato – building 1

Armando BELLANTE – XXXVI Cycle

Quantum algorithms for sparse recovery and machine learning

Supervisor: Prof. Stefano Zanero

Abstract:

Quantum computing is a novel computational paradigm that promises substantial speed-ups in a plethora of tasks that are computationally challenging for classical computers. Simultaneously, machine learning has experienced remarkable advancements in recent years, driven by breakthroughs in deep learning, increased computational power, and the availability of vast datasets. These advancements have significantly improved tasks such as image recognition, natural language processing, and autonomous systems.

In this PhD thesis, we explore faster quantum algorithms for learning problems and new classical algorithms for graph embeddings. This thesis is structured into three parts, with the main interleaving threads being quantum computation in the circuit model, machine learning and data analysis, and sparsity.

Part one discusses what quantum computers can do for sparse recovery problems, where the aim is to reconstruct a target vector using the sparsest combination of basis vectors from an overcomplete dictionary. First, we present quantum versions of the matching pursuit and orthogonal matching pursuit algorithms that achieve a polynomial speed-up over their classical counterparts under the assumption of an efficient classical-writable quantum-readable memory. Then, we define the novel problem of quantum sparse recovery - which consists of recovering the sparsest support of a target quantum state over an overcomplete set of other quantum states - and give conditions for which our quantum orthogonal matching pursuit algorithm efficiently recovers the optimal solution, without the need for a quantum memory.

Part two focuses on quantum algorithms for machine learning and data analysis, intending to speed up the training and classification steps of learning algorithms. Under the assumption of an efficient quantum memory, the thesis explores singular value decomposition-based problems and develops quantum algorithms to efficiently retrieve top singular values and vectors of matrices with good low-rank approximations, with applications in machine learning methods such as principal component analysis, correspondence analysis, and latent semantic analysis. Furthermore, it presents an efficient quantum classification algorithm based on a nearest neighbor/centroid algorithm and a norm-based outlier detection procedure, inspired by the

seminal eigenfaces approach for face recognition. Lastly, the practical potential of these fault-tolerant quantum algorithms is evaluated through numerical experiments and the discussion of an evaluation framework that can guide computer scientists and engineers toward assessing the potential of fault-tolerant quantum machine learning algorithms on real-world datasets and problems.

The third and last part of the thesis studies permutation-invariant graph embeddings, particularly the skew spectrum of graphs. The skew spectrum provides a time-efficient invariant embedding based on group theory and harmonic analysis of the symmetric group. On the road to developing a quantum group theoretic graph embedding, we stop and generalize this classical algorithm in two ways. First, we study a multi-orbit version of the skew spectrum algorithm, improving its completeness (the ability to map non-isomorphic graphs to different vectors) for more complex graph structures like multigraphs or graphs with node/edge attributes. Then, we extend the computation of the multi-orbit skew spectrum to the efficient computation of the spectrum of higher correlation functions, allowing a trade-off between computational efficiency and completeness. The efficient computation of the spectra is achieved through efficient Fourier transforms on the symmetric group that leverages a form of sparsity of the functions representing graphs, which are always constant over a large portion of the domain. We implemented and tested the three graphs invariant to show experimental evidence of the increased completeness.

In summary, this thesis contributes to advancing the field of quantum algorithms for sparse recovery and machine learning and developing strongly mathematically supported permutation-invariant graph embeddings.

PhD Committee

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